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Adaptive mixture filtering: an alternative to particle filtering?

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The Goal:

- Analytically approximate the filtering distributions of non-linear, non-Gaussian state space models

The Procedure:

(all distributions conditioned on $y_{1:t-1}$)

- Assume filtered posterior

$$p(x_{t-1}) = \sum_{j=1}^{J} p_{t-1,j} f(\theta_{t-1,j})$$

- Approximate the joint
  - by propagating posterior through the evolution equation

$$p(x_t, x_{t-1}) \approx \sum_{j=1}^{J} p_{t-1,j} p(x_t|\theta_{t-1,j}) f(\theta_{t-1,j})$$

- Extract and regenerate the prior
  - component means are spread out
  - component variances are small

$$p(x_t) \approx \sum_{j=1}^{J} p_{t-1,j} f(\theta'_{t,j})$$

- Approximate the joint, extract predictive
  - by propagating prior through the observation equation

$$p(y_t, x_t) \approx \sum_{j=1}^{J} p_{t-1,j} p(y_t|\theta'_{t,j}) f(\theta'_{t,j})$$

$$p(y_t) \approx \sum_{j=1}^{J} p_{t-1,j} g(\phi_{t,j})$$

- Approximate the posterior
  - adjust component probabilities

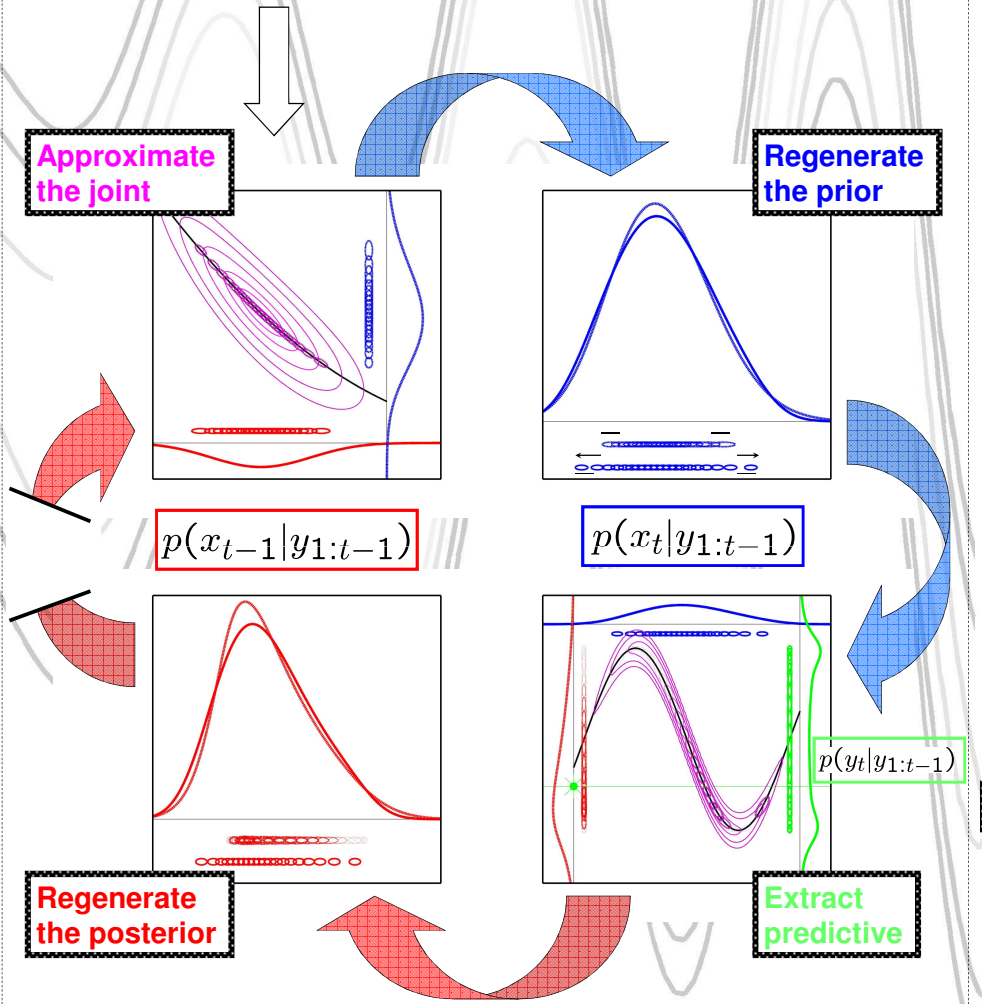
$$p(x_t|y_t) \approx \sum_{j=1}^{J} p'_{t,j} f(\theta'_{t,j})$$

- Regenerate the posterior
  - component probabilities are equal
  - component means are spread out
  - component variances are small

$$p(x_t|y_t) \approx \sum_{j=1}^{J} p_{t,j} f(\theta_{t,j})$$

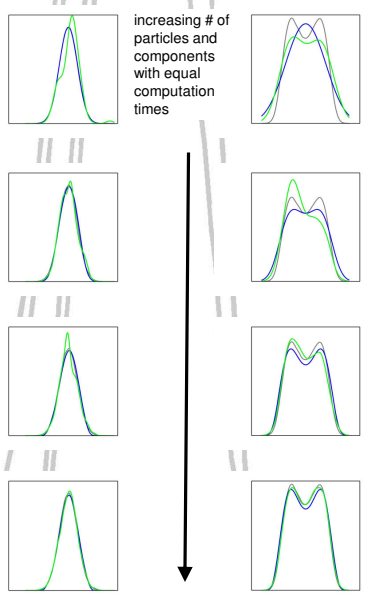
A non-linear Gaussian example:

$$\begin{aligned} y_t &= f_t(x_t) + \nu_t & \nu_t &\sim N(0, V_t) \\ x_t &= g_t(x_{t-1}) + \omega_t & \omega_t &\sim N(0, W_t) \\ x_0 &= \textstyle\sum_{j=1}^{J} p_{0,j} N(m_{0,j}, C_{0,j}) \end{aligned}$$



vs particle filtering:

$$\begin{aligned} f_t(x) &= x & f_t(x) &= x^2 \\ g_t(x) &= x & g_t(x) &= x - .2\text{atan}(x) \\ V &= 0.1 & V &= 0.1 \\ W &= 0.1 & W &= 0.1 \\ p(x_0) &\sim N(0,1) & p(x_0) &\sim N(0,1) \end{aligned}$$



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